Mouvement brownien branchant avec sélection

Soutenance de thèse de **Pascal MAILLARD**

effectuée sous la direction de Zhan SHI

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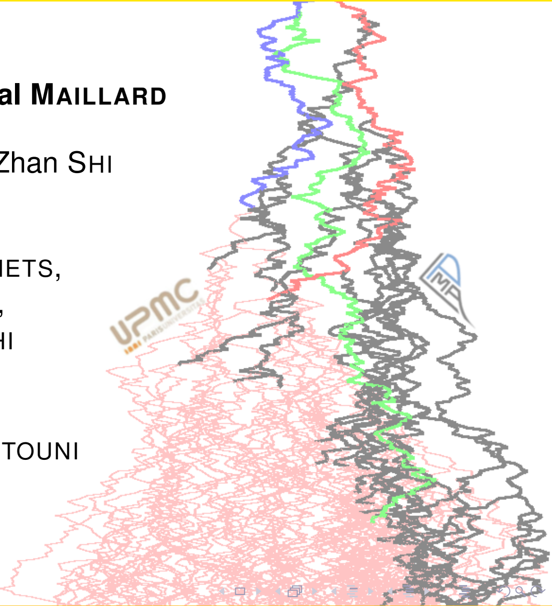

Université Pierre et Marie Curie
11 octobre 2012

Thesis structure

Introduction + 3 chapters:

1. The number of absorbed individuals in branching Brownian motion with a barrier
2. Branching Brownian motion with selection of the N right-most particles
3. A note on stable point processes occurring in branching Brownian motion

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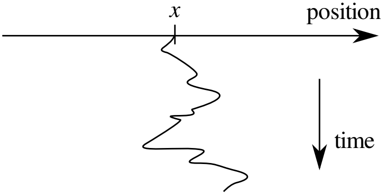
In this presentation: Chapters 1 and 2.

Outline

Branching Brownian motion (BBM)

Definition

- A particle performs standard Brownian motion started at a point $x \in \mathbb{R}$.



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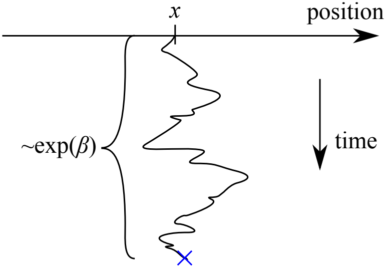

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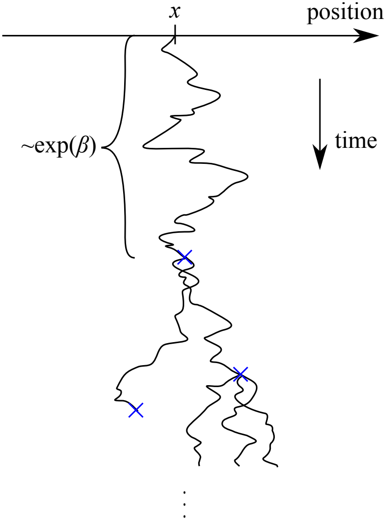

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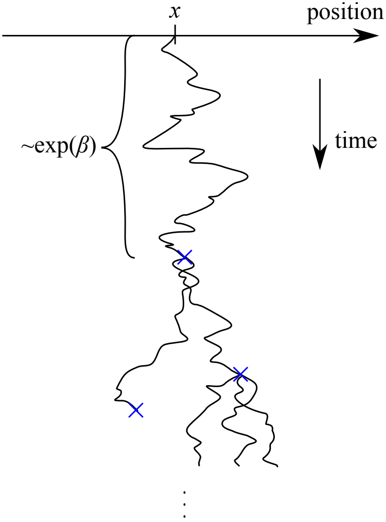
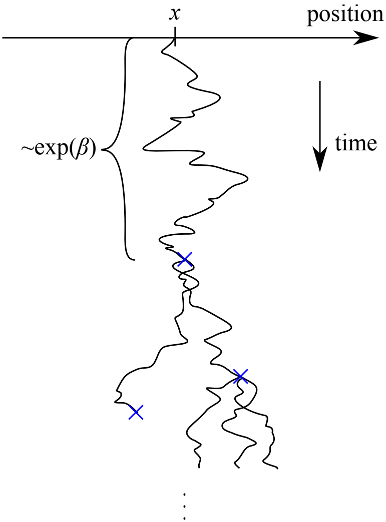
$\longrightarrow$ A **Brownian motion** indexed by a **tree**.



Branching Brownian motion (BBM) (2)

Context

- An example of a multitype branching process (type space: $\mathbb{R}$)



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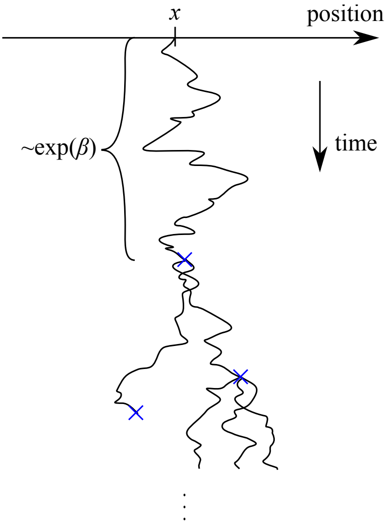
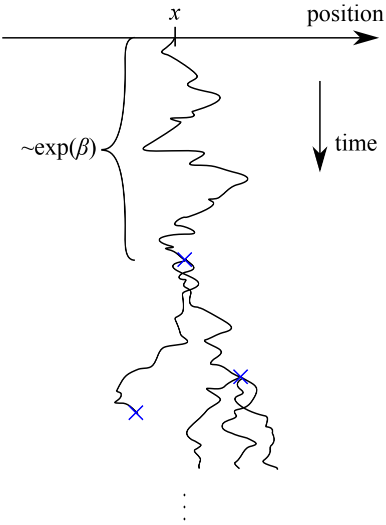
- Discrete counterpart: branching random walk



Branching Brownian motion (BBM) (2)

Context

- An example of a multitype branching process (type space: $\mathbb{R}$)
- Discrete counterpart: branching random walk
- Interpretations:
  - Model for an asexual population undergoing mutation (position = fitness)
  - Spin glass (with infinitely deep hierarchy)
  - Directed polymer on a tree
  - Prototype of a travelling wave



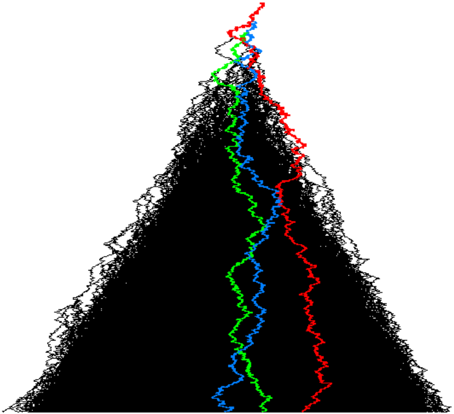
Branching Brownian motion (BBM) (3)

We always suppose $m := \mathbb{E}[L] - 1 > 0$.

Right-most particle

Let R_t be the position of the right-most particle. Then, as $t \to \infty$, almost surely on the event of survival,

$$\frac{R_t}{t} \to \sqrt{2\beta m}.$$



Picture by Éric Brunet

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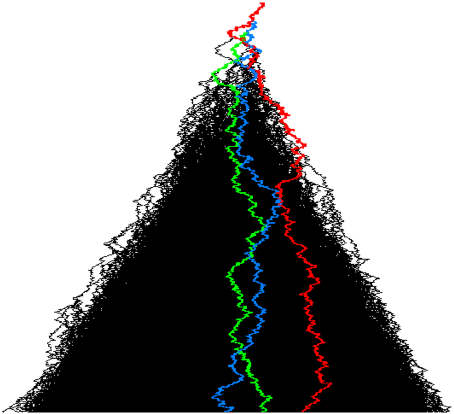
Convention

We will henceforth set $\beta = 1/(2m)$.



Picture by Éric Brunet

BBM $\longleftrightarrow$ FKPP

Let $g : \mathbb{R} \to [0,1]$ be measurable. Define

$$u(t,x) = \mathbb{E}_x\Big[\prod_{u\in\mathcal{N}_t} g(X_u(t))\Big].$$

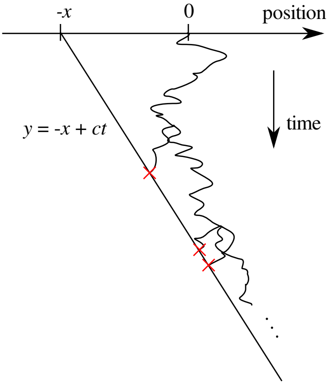
Then u satisfies the following partial differential equation:

Fisher–Kolmogorov–Petrovskii–Piskunov (FKPP) equation

$$\begin{cases} \partial_t u = \frac{1}{2}\partial_x^2 u + \beta(\mathbb{E}[u^L] - u) \\ u(0,x) = g(x) \quad \text{(initial condition)} \end{cases}$$

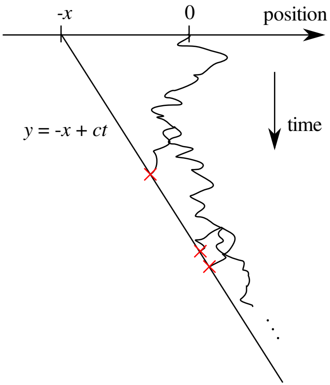
The prototype of a parabolic PDE admitting travelling wave solutions.

Selection



Two models of BBM with **selection**:

Selection



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1. **BBM with absorption**: Let $f(t)$ be a continuous function (the **barrier**). Kill an individual as soon as its position is less than $f(t)$ (*one-sided FKPP*).

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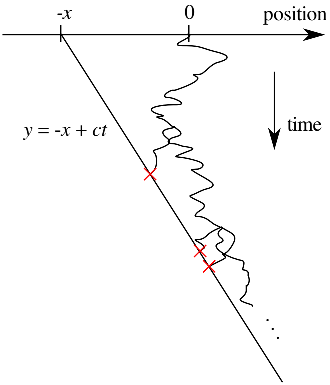
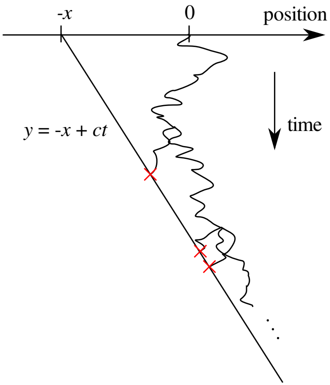


Two models of BBM with **selection**:

1. **BBM with absorption**: Let $f(t)$ be a continuous function (the **barrier**). Kill an individual as soon as its position is less than $f(t)$ (*one-sided FKPP*).
2. **BBM with constant population size (N-BBM)**: Fix $N \in \mathbb{N}$. As soon as the number of individuals exceeds N, kill the left-most individuals until the population size equals N (*noisy FKPP*).

Outline

Branching Brownian motion with absorption



We take $f(t) = -x + ct$ (*linear barrier*).
Vast literature, known results (sample):

- almost sure extinction $\Leftrightarrow c \geq 1$
  ($c = 1$: critical case
  $c > 1$: supercritical case)
- growth rates for $c < 1$.
- asymptotics for extinction probability for $c = 1 - \varepsilon$, ε small

We are interested in the number of absorbed individuals in the case $c \geq 1$ (question raised by D. Aldous).

Our results (critical case)

Let Z_x denote the number of individuals absorbed at the line $-x + ct$.

Theorem

Assume that $c = 1$ *and that* $\mathbb{E}[L(\log L)^2] < \infty$*. For each* $x > 0$*,*

$$\mathbb{P}(Z_x > n) \sim \frac{xe^x}{n(\log n)^2}, \quad \text{as } n \to \infty.$$

If, furthermore, $\mathbb{E}[s^L] < \infty$ *for some* $s > 1$*, then*

$$\mathbb{P}(Z_x = \delta n + 1) \sim \frac{xe^x}{\delta n^2(\log n)^2} \quad \text{as } n \to \infty,$$

where δ *is the span of* $L - 1$*.*

Our results (supercritical case)

Theorem

Assume that $c > 1$ *and that* $\mathbb{E}[s^L] < \infty$ *for some* $s > 1$. *Let* $\lambda_c < \overline{\lambda_c}$ *be the roots of the equation* $\lambda^2 - 2c\lambda + 1 = 0$ *and define* $d = \overline{\lambda_c}/\lambda_c$.
There $\exists K = K(c, L) > 0$, *such that for all* $x > 0$,

$$\mathbb{P}(Z_x = \delta n + 1) \sim \frac{K(e^{\overline{\lambda_c} x} - e^{\lambda_c x})}{n^{d+1}} \quad \text{as } n \to \infty.$$

Other studies

Addario-Berry and Broutin (2011), Aïdékon (2010): Less precise tail estimates ($c = 1$).

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In contrast to the above papers, our proofs are entirely analytic.
Strategy: derive asymptotics on the generating function of Z_x near its singularity 1 (following an idea of R. Pemantle's).

The number of absorbed individuals

Theorem (Neveu, 1988)

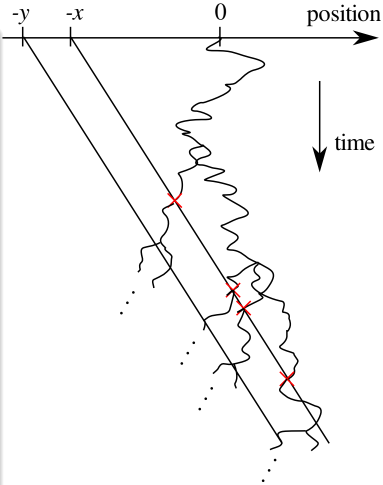
$(Z_x)_{x\geq 0}$ is a continuous-time Galton–Watson process. The infinitesimal generating function $a(s) = \mathrm{d}\mathbb{E}[s^{Z_x}]/\mathrm{d}x$ admits the decomposition

$$a = -\psi' \circ \psi^{-1},$$

where ψ is an FKPP travelling wave of speed c, i.e.

$$\tfrac{1}{2}\psi''(s) - c\psi'(s) + \beta(\mathbb{E}[s^L] - s) = 0,$$

and $\psi(x) \uparrow 1$, as $x \to \infty$.



Tail asymptotics $c = 1$

Follow from a Tauberian theorem and the following lemma:

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Proof of lemma:

- Solve two-dimensional ODE satisfied by (ψ', ψ)
- Use known asymptotic: $1 - \psi(x) \sim Cxe^{-x}$ as $x \to \infty$.

Asymptotics on density ($c \geq 1$)

Derive asymptotics of $a(s)$ near $s = 1$ in the complex plane and use transfer theorems by Flajolet and Odlyzko.

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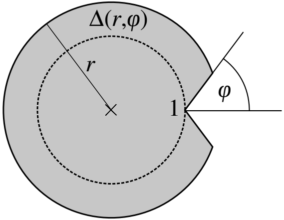
Derive asymptotics of $a(s)$ near $s = 1$ in the complex plane and use transfer theorems by Flajolet and Odlyzko.

To this end,

- show that $a(s)$ can be analytically extended to a region $\Delta(r, \varphi)$,
- analyse its asymptotic behaviour near the point $s = 1$ inside $\Delta(r, \varphi)$.



Asymptotics on $a(s)$ near $s = 1$

Theorem

For every $\varphi \in (0, \pi)$ there exists $r > 1$, such that $a(s)$ possesses an analytical extension to $\Delta(\varphi, r)$. Moreover, as $1-s \to 1$ in $\Delta(\varphi, r)$, the following holds.

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$$a(1-s) = -s + \frac{s}{\log \frac{1}{s}} - s\frac{\log\log \frac{1}{s}}{(\log \frac{1}{s})^2} + \frac{Ks}{(\log \frac{1}{s})^2} + o\left(\frac{s}{(\log \frac{1}{s})^2}\right).$$

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- *If $c > 1$, then $\exists K = K(c, L) \neq 0$ and a polynomial $h(s)$, such that*
  if $d \notin \mathbb{N}$: $\quad a(1-s) = -\lambda_c s + h(s) + Ks^d + o(s^d)$,
  if $d \in \mathbb{N}$: $\quad a(1-s) = -\lambda_c s + h(s) + Ks^d \log s + o(s^d)$.

Proof: Main idea

As before, write two-dimensional ODE satisfied by (ψ', ψ) in a subset of the complex plane. Changing coordinates leads to the classic

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Briot–Bouquet equation

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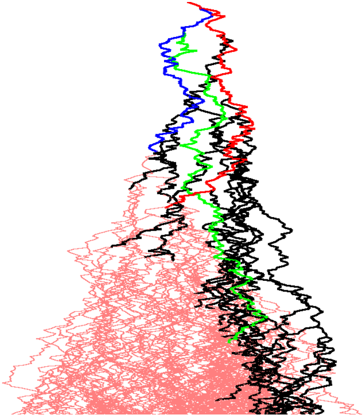
Note. Major technical difficulty in the proofs: justifying the coordinate changes.

Outline

BBM with constant population size



Picture by Éric Brunet

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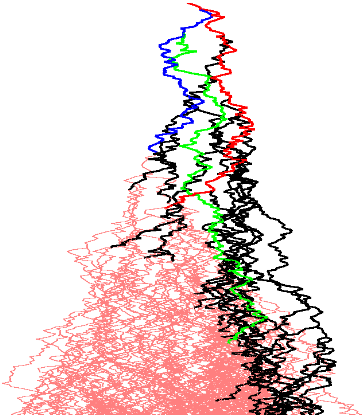

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Nevertheless: A fairly detailed heuristic picture due to physicists: Brunet and Derrida (1997-2004) with Mueller and Munier (2006-2007)

Heuristic picture of N-BBM (BDMM 06)

- Meta-stable state: speed $c_N^{\text{det}} = \sqrt{1 - \pi^2/\log^2 N}$, empirical measure seen from the left-most particle approximately proportional to $\sin(\pi x/\log N)e^{-x}\mathbf{1}_{(0,\log N)}(x)$, diameter $\approx \log N$.

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Real speed of the system is approximately

$$c_N = \sqrt{1 - \frac{\pi^2}{a_N^2}} = c_N^{\text{det}} + \frac{3\pi^2 \log\log N + o(1)}{\log^3 N},$$

and $O(1/\log^3 N)$ fluctuations.

Main result

Order the individuals according to position: $X_1(t) > X_2(t) > \dots$
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Suppose $\mathbb{E}[L^2] < \infty$ *and at time* 0*, there are* N *particles distributed independently in* $(0, a_N)$ *according to density proportional to* $\sin(\pi x/a_N)e^{-x}$*. Then, for every* $\alpha \in (0, 1)$*,*

$$\left(X_{\alpha N}(t\log^3 N) - c_N t\log^3 N\right)_{t\geq 0} \overset{fidis}{\Longrightarrow} (L_t + x_\alpha)_{t\geq 0}.$$

Here, $(L_t)_{t\geq 0}$ *is a (pure-jump) Lévy process with* $L_0 = 0$ *and Lévy measure the image of* $\pi^2 x^{-2}\mathbf{1}_{x>0}\,\mathrm{d}x$ *by the map* $x \mapsto \log(1 + x)$*.*

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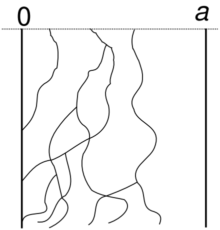
Proof idea: Approximate the N-BBM by BBM with a certain (random) absorbing barrier, called the B-BBM.

The B-BBM

a: Position of a second barrier (idea from BBS (2010)).
Add drift $-c$, with $c = \sqrt{1 - \pi^2/a^2}$.
A: Determines number of particles ($N \approx 2\pi e^{A+a}/a^3$).
Let first a, then A go to ∞.



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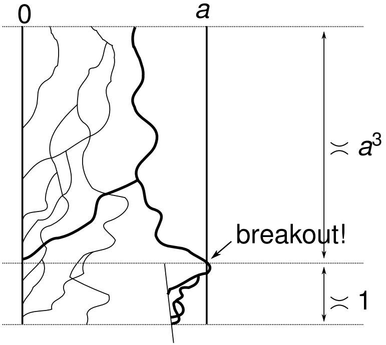
Breakout when $W > \varepsilon e^A$, ε small.



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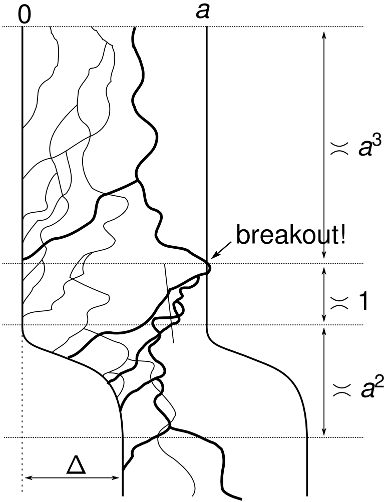
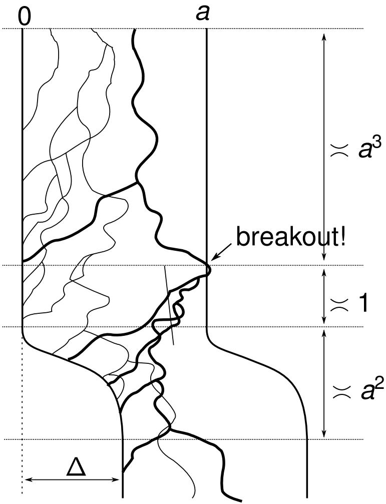
After breakout, move barrier smoothly by random amount Δ.



The B-BBM (continued)

Three details:

1. Particles that hit *a* and have few descendants are important: compensator for the limiting Lévy process.



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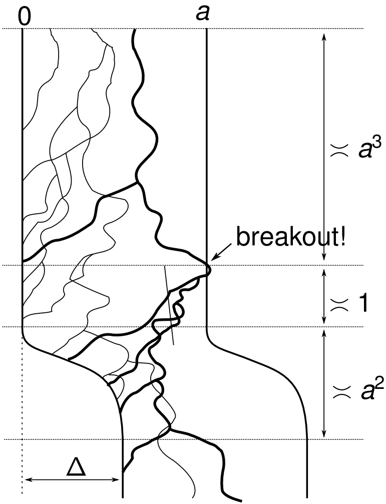
1. Particles that hit *a* and have few descendants are important: compensator for the limiting Lévy process.
2. B-BBM until the first breakout = spine + BBM (weakly) conditioned not to hit *a* (Doob transform of BBM).



The B-BBM (continued)

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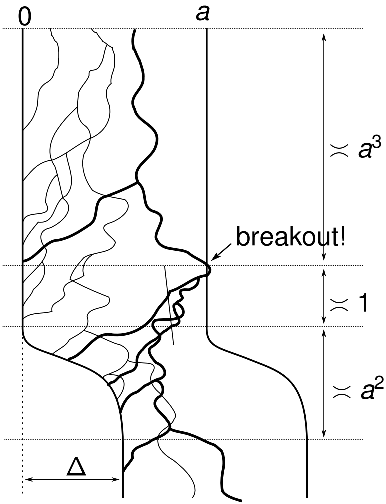
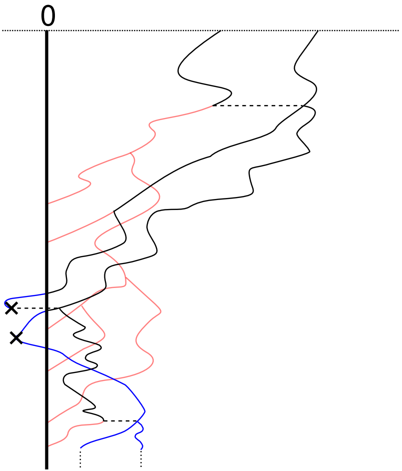
2. B-BBM until the first breakout = spine + BBM (weakly) conditioned not to hit a (Doob transform of BBM).
3. Shape of barrier given by a family $(f_\Delta)_{\Delta\geq 0}$ of explicitly given, smooth, increasing functions with $f_\Delta(0) = 0$ and $f_\Delta(+\infty) = \Delta$.



B-BBM ↔ *N*-BBM

First idea: couple both processes.

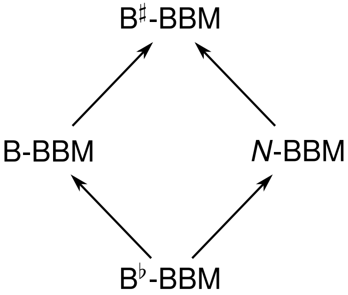
- **black** particles: present in B-BBM **and** *N*-BBM,
- red particles: present in B-BBM but **not** in *N*-BBM,
- blue particles: present in *N*-BBM but **not** in B-BBM.



Problem

Dependencies between particles too difficult to handle.

The solution



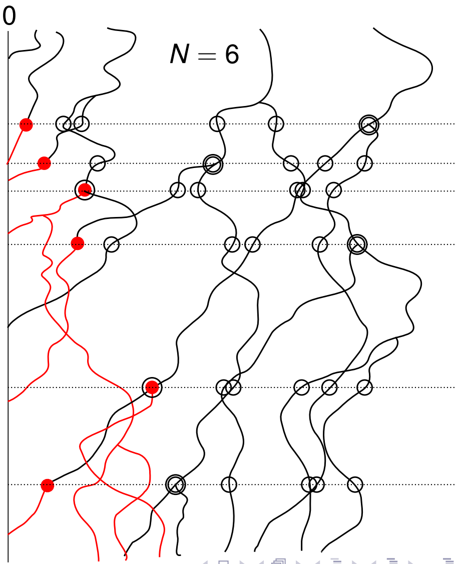
Introduce two auxiliary particle systems: The B$^\flat$-BBM and the B$^\sharp$-BBM (stochastically) bound the N-BBM (and the B-BBM) from below and above (in the sense of stochastic order on the empirical measures).

Bounding the N-BBM from below: The $B^\flat$-BBM

Kill a particle

- whenever it hits 0 **or**
- whenever it has N particles to its right (red particles).

$\Longrightarrow$ more particles are being killed than in N-BBM.



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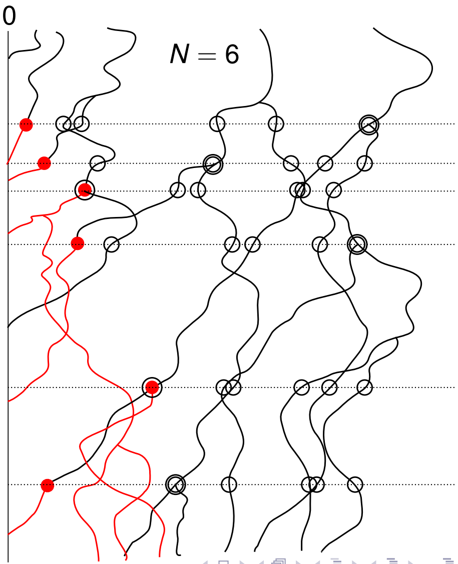
At timescale $\log^3 N$, number of red particles stays negligible.

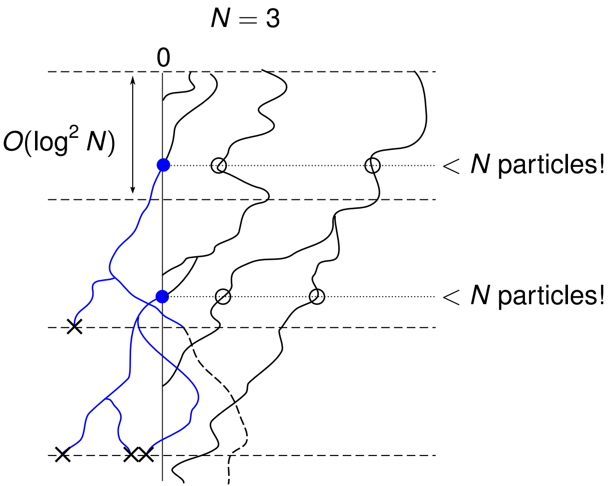


Bounding the N-BBM from above: The B$^\sharp$-BBM

Kill a particle whenever it (at the same time)

- hits 0 **and**
- has N particles to its right.

A particle survives temporarily (blue particles) if it has less than N particles to its right the moment it hits 0.



Outline

N-BBM $\longleftrightarrow$ noisy FKPP

Noisy FKPP equation

$$\begin{cases} u(t,x) : \mathbb{R}_+ \times \mathbb{R} \to [0,1] \\ \partial_t u = \partial_x^2 u + u(1-u) + \sqrt{\varepsilon u(1-u)}\,\dot{W} \\ u(0,x) = \mathbf{1}_{(x<0)} \qquad \text{(IC)} \end{cases}$$

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- Admits travelling wave solutions with same phenomenology as N-BBM ($N \simeq \varepsilon^{-1}$), cf Mueller, Mytnik and Quastel (2010)
- Dual to BBM with particles coalescing at rate ε.
  $\longrightarrow$ density-dependent selection

Empirical measure

Known: Empirical measure of N-BBM seen from the left-most particle is an ergodic Markov process.

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Open problem

Show that stationary probability converges as $N \to \infty$ to the Dirac-measure in $xe^{-x}\,\mathrm{d}x$.

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$\longrightarrow$ ongoing work with J. Berestycki and M. Jonckheere.

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Ongoing work joint with Jean Bérard: Consider N-BRW where at each time step, particles split into two and children jump according to the law of a random variable $X \geq 0$, with $\mathbb{P}(X > x) \sim x^{-\alpha}$, $\alpha > 0$. Keep only the N right-most particles at every time step.

Right scaling: space by $(N \log N)^{1/\alpha}$, time by $\log N$.

Other open questions

- Speed of the system
- Genealogy
- Inhomogeneous media
- ...